6.6.1.3 Different demand rate for different cycles but total demand for the planning period is known.

In the first case, the demand rate is constant for each time period cycle, but the demand rate may vary in time period cycle. Let's assume that $D_1, D_2, D_3, \ldots\ldots, D_n$ is the demand during the time period $t_1, t_2, t_3, \ldots\ldots, t_n$. Suppose, each time a fixed quantity say q is to be ordered, then the number of orders in total time period T (= $t_1 + t_2 + t_3, \ldots\ldots + t_n$) will be $\frac{D}{Q}$.

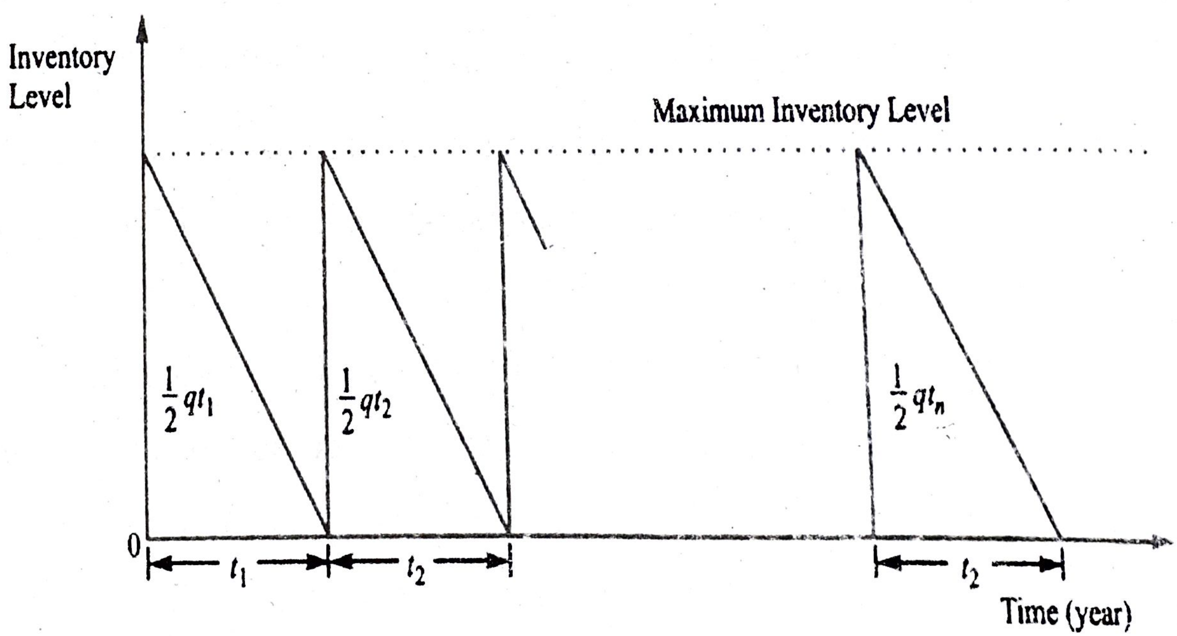

Figure 6.6 Different demand rate for different cycles but total demand for the planning period is known

Now, the carrying cost for total time period cycle,

$$= \frac{1}{2} Q \cdot t_1 C_h + \frac{1}{2} Q \cdot t_2 C_h + \ldots\ldots\ldots\ldots + \frac{1}{2} Q \cdot t_n C_h = \frac{1}{2} Q \cdot T \cdot C_h$$

Ordering cost, $= n \cdot C_o = \frac{D}{Q} \cdot C_o$

Since, the order costs decreases and holding costs increases, when the production quantity (Q) increases, therefore, a minimum TVC occurs when these two costs are equal. *i.e,*

$$= \frac{1}{2} Q \cdot T \cdot C_h = \frac{D}{Q} \cdot C_o \qquad \therefore Q = \sqrt{\frac{2C_0 D}{C_h}}$$

From this equation, finding out the **TVC,** $= \sqrt{2\frac{D}{T} C_0 C_h}$

6.6.2. Inventory Models With Constant Demand and Shortage is Allowed

In the previous subsection, we discussed the inventory models with assumptions that no shortage is allowed. So, we have not involved trade-off between ordering cost and carrying cost. In this section, we will consider that shortage is allowed. Following are the two cases as below;

1. Demand is constant and the time of reorder cycle time is variable.
2. Demand is constant and the time of reorder cycle is fixed.
3. Gradual supply and shortage is allowed.

6.6.2.1 Demand is constant and the time of reorder cycle time is variable.

This model is based on the assumptions that are made in the case of sub-section **6.6.1.1** except shortage is allowed. The cost of shortage in this case is assumed to be directly proportional to the average number of units short. If shortage occurs, two scenarios will occur: First one, due to shortage customers will not purchase and sales will be lost. Otherwise customers will wait to receive an order from the supplier and such backorders are filled as stock is available.

In second scenario, backorder cost is involved. This backorder cost involves cost of keeping back log records, cost of shipping items to the customers, loss of goodwill.

Now, assume some notations.

t_1 = time between receipt of the inventory up to last stock of the inventory lasts

t_2 = time during shortage exists.

t = total cycle time

R = Maximum Shortage

Q = Quantity ordered

M = Surplus quantity after satisfying demand of shortage = Q – R.

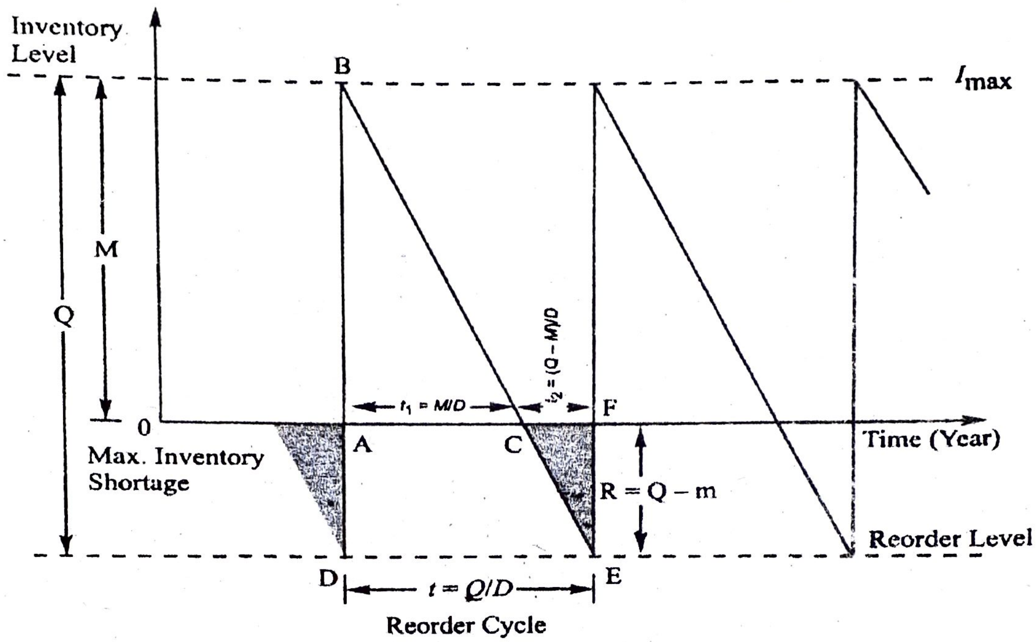

Figure 6.7 Demand is constant and the time of reorder cycle time is variable

As shown in Fig 6.7, every time when Q amount of quantity is ordered, from that R amount of shortage is first of all satisfied. The remaining quantity M is placed in inventory as the surplus and it is used for satisfying the demand during the next cycle.

It is noticeable point that 'R' quantity from ordered 'Q' quantity is always in shortage in every cycle. So, that 'R' amount of quantity never fetches inventory cost.

Also, expressing the quantity Q, M and R in terms of their corresponding time as below:

$$Q = D \cdot t; \qquad M = D \cdot t_1; \qquad R\,(Q - M) = D \cdot t_2$$

So, total cost is the sum of ordering cost, holding cost and shortage cost, we calculate each component.

$$\textbf{Ordering cost} = \frac{D}{Q} \cdot C_0$$

HOLDING COST CALCULATION

Now, for finding out the average inventory holding cost, we need to know the value of average inventory.

$$I_{avg} = \frac{(I_{avg})t_1 + (I_{avg})t_2}{t} = \frac{\frac{M}{2}t_1 + 0 \cdot t_2}{t}$$

Putting the value of t_1 and t into the above equation, we get; $I_{avg} = = \frac{\frac{M}{2} \cdot (M/D)}{Q/D} = \frac{M^2}{2Q}$

So, the carrying cost $= \frac{M^2}{2Q} \cdot C_h$

SHORTAGE COST CALCULATION

Now, finding the shortage cost, by dividing the area under the shortage triangle *i.e.,* Δ *CEF,* by total time of the cycle, t. So, shortage cost,

$$= \frac{1}{2} R \cdot t_2 \ldots\ldots\ldots\ldots\ldots (4)$$

Putting the value of t_2 into the above equation, we get;

$$= \frac{1}{2}\frac{(Q-M)(Q-M)}{D \cdot t}$$

$$= \frac{1}{2}\frac{(Q-M)^2}{D \cdot t}$$

$$= \frac{1}{2}\frac{(Q-M)^2}{Q}$$

So, the Shortage cost, $= \frac{1}{2}\frac{(Q-M)^2}{Q} C_s$

$$\text{So, TVC} = \frac{D}{Q} \cdot C_0 + \frac{M^2}{2Q} \cdot C_h + \frac{1}{2}\frac{(Q-M)^2}{Q} C_s$$

Since, all costs and demand is constant, TVC is the function of Q and M, so, in order to have optimal order size (Q*) and the optimal shortage level (R*), differentiating the TVC with respect to Q and M; equating those equations to zero and solving simultaneously we get:

$$\textbf{EOQ} = Q^* = \sqrt{\frac{2C_0D(C_h + C_s)}{C_hC_s}} \quad \& \quad \textbf{Optimal Stock Level} = M^* = \sqrt{\frac{2C_0DC_s}{C_h(C_h + C_s)}}$$

Putting this values in TVC, optimal TVC= TVC* = $\sqrt{2C_0DC_h \frac{C_s}{(C_h + C_s)}}$

∴ Optimal Shortage level, R* = Q* - M*.

SOLVED PROBLEMS

Problem 6.18: Following are the details available information for a company regarding a particular product:

Annual demand = 15,000 units; Ordering cost = Rs. 10 per order;

Cost = Rs. 20 per product; Inventory holding cost = 20% of cost per year.

Shortage cost = 25% of value of inventory.

Calculate:

(i) Economic order quantity;

(ii) Allowable shortage of quantity;

(iii) Maximum inventory level

(iv) Total variable cost.

Solution:

Here,

D = 15,000 units; Co = Rs. 10 per order; C = Rs. 20;

C_h = Rs. 4 per unit per year. C_s = Rs. 5 unit per year.

1. Economic order quantity, $Q^* = \sqrt{\dfrac{2C_0D(C_h + C_s)}{C_hC_s}} = \sqrt{\dfrac{2(15000)(10)(4+5)}{4 \cdot 5}}$ = 368 units.
2. Allowable shortage of quantity, $R^* = Q^* \cdot \left(\frac{C_h}{C_h+C_s}\right) = 368\left(\frac{4}{9}\right) = 164$ units
3. Maximum inventory level, $M^* = 368 - 164 = 204$ units.
4. $TVC = \sqrt{2C_0DC_h \dfrac{C_s}{(C_h + C_s)}} = \sqrt{\dfrac{2(15000)(10) \cdot 4 \cdot 5}{(4+5)}}$ = Rs. 817.

Problem 6.19: In the above example, consider that if no shortage is allowed, then calculate:

(i) Economic order quantity and

(ii) Also, check whether the adopting the shortage condition is in favour of company or not.

Solution:

(i) Economic order quantity if no shortage is there, $Q = \sqrt{\dfrac{2C_0D}{C_h}} = \sqrt{\dfrac{2(10)(15000)}{4}}$ = 274 units

(ii) Now, checking for the cost when no shortage is permitted;

$$TVC = \sqrt{2DC_0C_hD} = \sqrt{2(15000)(10)(4)} = \text{Rs. } 1096$$

If company adopts the shortage condition, than saving = Rs. (1096 - 817) = Rs. 279.

EXERCISE 6.16: A particular product is to be supplied at a rate of 200 units per day. Ordering cost is Rs. 50 and inventory holding cost is Rs. 2 per day. The penalty for not fulfilling the demand is Rs. 10 per unit per day. Find the optimum quantity and reorder time.

EXERCISE 6.17: An item has demand of 9,000 units per year. The cost of ordering is Rs. 100 and holding cost is Rs. 2.4 per year as well as shortage cost is Rs. 5 per unit per year. Find optimum quantity and reorder time.

6.6.2.2 Demand is constant and the time of reorder cycle is fixed.

Now, when cycle time is fixed, say t, after every t time inventory is supplied. Here, time is also fixed, so the Q will also become constant, if demand is constant. So, the total cost is only function of M only. So, minimum TVC depends only on M.

$$\text{So, TVC }(M) = \frac{M^2}{2Q} \cdot C_h + \frac{1}{2}\frac{(Q-M)^2}{Q} C_s$$

Differentiating above equation with respect to M and equating it with zero, we get:

$$M^* = \frac{C_s}{(C_h + C_s)} \cdot Q$$

By substituting the value of M in TVC, the minimum cost is found out,

$$TVC^* = \frac{C_s \cdot C_h}{(C_h + C_s)} Q$$

SOLVED PROBLEMS

***Problem 6.20:** A bus repairing garage is repairing at the rate of 20 buses per day. He has to pay a penalty of Rs. 100 per bus per day for not fulfilling the demand. Carrying cost of bus is Rs. 1200 per month. What should be the inventory level at the beginning of each month? Also, calculate minimum total variable cost.*

Solution:

Here,

$d = 20$ per day; C_s = Rs. 100 per engine per day;

C_h = Rs. 1200 per month, $\therefore C_h$ = Rs. 40 per day

$Q = 20 * 30 = 600.$

$$M^* = \frac{C_s}{(C_h + C_s)} \cdot Q = \frac{100}{(100+40)} * 600 = 428 \text{ buses/month}$$

$$TVC^* = \frac{C_s \cdot C_h}{(C_h + C_s)} * Q = \frac{100 \cdot 40}{(100+40)} * 600 = \text{Rs. } 17142 \text{ per month.}$$

PRACTICE PROBLEM

EXERCISE 6.18: A company is producing cotton units at the rate of 50 per day. He has to pay a penalty of Rs. 10 per bus per day for not fulfilling the demand. Carrying cost of bus is Rs. 16 per unit per month. What should be the optimum inventory level at the beginning of each month (30 days)? Also, calculate minimum total variable cost.

6.6.2.3 Inventory models with Gradual Supply and Shortage is allowed.

This model has following inventory system as shown in figure. In this kind of model,

- During time t_1, the ***inventory is accumulated*** at the rate of *p-d* and maximum inventory is at the end time t_1 is Q_1,
- During t_2 time, that ***inventory is used*** at the rate of *d*.
- During time t_3, ***shortage is accumulated*** at the demand rate *(D)* and maximum shortage allowed is Q_2.
- During time end of time t_4, ***production is started and therefore shortage reduces*** at the rate of $(p\text{-}d) \cdot t_4$.

So, total cycle time is $t = t_1 + t_2 + t_3 + t_4$.

Now, following is the **inventory accumulation and consumption** during each time period:

- During time t_1, maximum inventory accumulated Q_1 is given by, $Q_1 = (p\text{-}d)\, t_1$
- During time t_2 the Q_1 inventory used at rate of d, so, Q_1 is given by, $Q_1 = d \cdot t_2$
- During time t_3 shortage accumulated in quantity, Q_2 is given by, $Q_2 = d \cdot t_3$
- During time t_4 shortage of Q_2 is reduced at the rate of *(p-d)*, $Q_2 = (p\text{-}d) \cdot t_4$

This time can be written (in terms inventory) as follows:

$t_1 = \dfrac{Q_1}{p-d}$	$t_2 = \dfrac{Q_1}{d}$	$t_3 = \dfrac{Q_2}{d}$	$t_4 = \dfrac{Q_2}{p-d}$

So, total production cycle time, $\dfrac{Q_1}{p-d} + \dfrac{Q_1}{d} + \dfrac{Q_2}{d} + \dfrac{Q_2}{p-d} + \dots\dots\dots$

As we know that, the total variable cost is the sum of ordering cost (when order is placed), holding cost (during t_1+t_2 time) and shortage cost (during $t_3 + t_4$ time). So, finding each component.

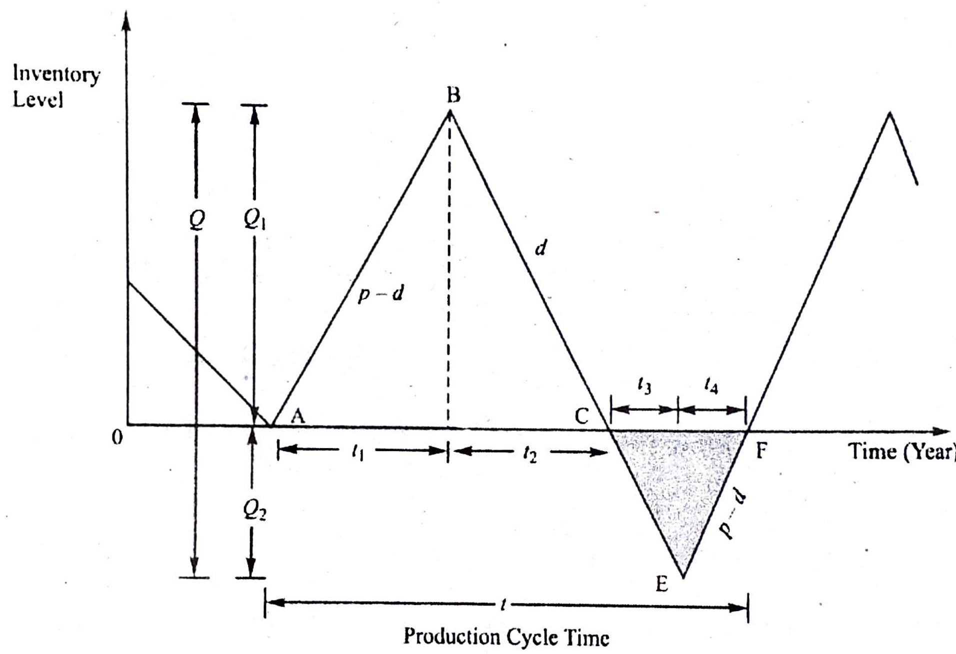

Figure 6.8 Inventory models with Gradual Supply and Shortage is allowed

Component 1: Ordering cost

Assuming $D = d$, the number of order $= \dfrac{d}{Q}$ $\therefore$ **Ordering cost** $= \dfrac{d}{Q} \cdot C_o$

Component 2: Carrying cost

Now, Carrying cost is the cost occurring during t_1 and t_2.

So, average inventory to be carried out during time $t_1 + t_2 = \dfrac{1}{2}\dfrac{Q_1(t_1+t_2)}{t}$

$$\therefore \textbf{Carrying Cost} = \frac{1}{2}\frac{Q_1(t_1+t_2)}{t} \cdot C_h$$

Component 3: Shortage cost

Now, Shortage cost is the cost occurring during t_3 and t_4.

So, the average shortage during total time $t_3 + t_4$, $= \dfrac{1}{2}\dfrac{Q_2(t_3+t_4)}{t}$

$$\therefore \textbf{Shortage Cost} = \frac{1}{2}\frac{Q_2(t_3+t_4)}{t} \cdot C_s$$

$$\text{So, total variable cost} = \frac{d}{Q} \cdot C_o + \frac{1}{2}\frac{Q_1(t_1+t_2)}{t} \cdot C_h + \frac{1}{2}\frac{Q_2(t_3+t_4)}{t} \cdot C_s$$

From above, we can write that TVC is function of Q, Q_1 and Q_2.

Practically it is difficult to differentiate the above equation of three variables and then solving simultaneously. So, we convert the equation in terms of Q and Q_1 using total production cycle time.

From the equation of total time, $t = \frac{Q_1}{p-d} + \frac{Q_1}{d} + \frac{Q_2}{d} + \frac{Q_2}{p-d}$

$$= Q_1\left(\frac{1}{p-d} + \frac{1}{d}\right) + Q_2\left(\frac{1}{p-d} + \frac{1}{d}\right) = \frac{p}{d(p-d)}(Q_1 + Q_2).$$

As we know that (See the *figure*), the total Q, is of Q_1, Q_2 and quantity used during time t_1 and t_4.

So, the total $Q = Q_1 + Q_2 + d \cdot t_1 + d \cdot t_4$.

$$= Q_1 + Q_2 + d\left(\frac{Q_1}{p-d} + \frac{Q_2}{p-d}\right)$$

$$= Q_1 + Q_2 + d \cdot \frac{Q_1 + Q_2}{p-d}$$

$$= (Q_1 + Q_2)\left(1 + \frac{d}{p-d}\right)$$

$$Q = (Q_1 + Q_2)\frac{p}{p-d} \qquad \therefore (Q_1 + Q_2) = Q \cdot \frac{p-d}{p}$$

So, putting this value in the above equation of time t, we get:

$$t = \frac{p}{d(p-d)}(Q_1 + Q_2)$$

$$= \frac{Q \cdot p}{d(p-d)} \frac{p-d}{p} \Rightarrow t = \frac{Q}{d}$$

So, total variable cost, $= \frac{d}{Q} \cdot C_o + \frac{1}{2}\frac{Q_1(t_1+t_2)}{t} C_h + \frac{1}{2}\frac{Q_2(t_3+t_4)}{t} . C_s$

$$= \frac{d}{Q} \cdot C_o + \frac{1}{2}\frac{Q_1}{t}\left(\frac{Q_1}{p-d} + \frac{Q_1}{d}\right) C_h + \frac{1}{2}\frac{Q_2}{t}\left(\frac{Q_2}{p-d} + \frac{Q_2}{d}\right) C_s$$

$$= \frac{d}{Q} \cdot C_o + \frac{1}{2t} Q_1^2 \left(\frac{1}{p-d} + \frac{1}{d}\right). C_h + \frac{1}{2t} Q_2^2 \left(\frac{1}{p-d} + \frac{1}{d}\right). C_s$$

$$= \frac{d}{Q} \cdot C_o + \frac{1}{2t} Q_1^2 \left(\frac{p}{d(p-d)}\right). C_h + \frac{1}{2t} Q_2^2 \left(\frac{p}{d(p-d)}\right). C_s$$

$$TVC (Q, Q_2) = \frac{d}{Q} \cdot C_o + \frac{1}{2Q}\frac{p}{p-d} (Q_1^2 . C_h + Q_2^2 . C_s) \qquad (\because t = \frac{Q}{d})$$

$$= \frac{d}{Q} \cdot C_o + \frac{1}{2Q}\frac{p}{p-d}\left[\{Q \cdot \left(\frac{p-d}{p}\right) - Q_2.\}^2 Ch + Q_2^2 . Cs\right]$$

Now, differentiating the above equation, with respect to the variables, we get:

Optimal Production lot size, $Q^* = \sqrt{\frac{2DC_o(C_h+C_s)}{C_hC_s}\left(\frac{p}{p-d}\right)}$

Optimum level of Shortage, $Q_2^* = Q^* \left(\frac{p-d}{p}\right)\left(\frac{C_h}{C_h+C_s}\right)$ *and*

$TVC^* = \sqrt{2DC_oC_h\left(\frac{C_s}{C_s+C_h}\right)}$ **(GTU – Dec. 2014)**

SOLVED PROBLEM

Problem 6.21: Determine the economic batch quantity and number of shortages for a particular item produced by a company having production of 30,000 annually against monthly demand of 1,500 units. The cost of set-up is Rs. 500 and the holding cost of each unit is Rs. 1.8 per year. Also, calculate total cycle time and total variable cost. The shortage cost of each unit is Rs. 25 per month per unit.

Solution:

Here, the demand as well as the supply is gradual. So, we use the formula of section 6.6.2.3.

p= 30,000 units per annum.

d (=D) = 1500 units per month = 18,000 units per annum

C_o = Rs. 500 per set-up

C_h = Rs. 1.80 per year per unit

C_s = Rs. 25 per month = Rs. 300 per year.

Optimum batch quantity, Q* $= \sqrt{2DC_o\left(\frac{p}{p-d}\right)\left(\frac{C_h+C_s}{C_h \cdot C_s}\right)}$

$= \sqrt{2(18,000)(500)\left(\frac{30000}{30000-18000}\right)\left(\frac{1.8+300}{1.8 \cdot 300}\right)}$

= 5015 units.

Optimal number of shortages = Q_2^* $= Q^*\left(\frac{p-d}{p}\right)\left(\frac{C_h}{C_h+C_s}\right)$

$= 5015 * \left(\frac{30000-18000}{30000}\right)\left(\frac{1.8}{1.8+300}\right)$ = 12 units.

Total cycle time, $t = \frac{Q*}{d} = \frac{5015}{1500}$ = 3 months 11 days.

TVC* = $\sqrt{2DC_oC_h\left(\frac{C_s}{C_s+C_h}\right)} = \sqrt{2(18,000)(1.8)(500)\left(\frac{300}{300+1.8}\right)}$ = Rs. 5676.

PRACTICE PROBLEM

EXERCISE 6.19: Determine the economic batch quantity and number of shortages for a particular item produced by a company having production of 24,000 annually against monthly demand of 1,000 units. The cost of set-up is Rs. 400 and the holding cost of each unit is Rs. 1.8 per year. Also, calculate total cycle time and total variable cost. The shortage cost of each unit is Rs. 20 per year per unit.

6.6.3. Multi-Item Inventory Control Models With Constraints

Still we have dealt with only one inventory item, but in routine production, it is not possible. There are many constraints such as floor, investment, average inventory control level and number of constraints.

In this section, we will discuss each constraint for multi-item. Following are the notations and assumptions in this section:

Assumptions:

1. Production and supply is instantaneous.
2. Demand is uniform and deterministic.
3. No shortage is possible.

Notations: (Suffix i represents for different inventory items, $i = 1, 2, 3 \ldots n$)

n = number of items to be controlled

D_i = annual demand for i^{th} item

Q_i = Order quantity for i^{th} item

C_i = Unit Price cost of i^{th} item

6.6.3.1 EOQ model with Space constraints.

Suppose, f_i is the floor area required for each item, then their total must be less than the total available floor area, say W. It means that even all items with order quantity is available, even than floor area must be sufficient to accommodate all. So, TVC minimum will depends on two factors: Ordering cost and inventory holding costs.

So, we can write, min TVC = $\sum_{i=1}^{n}\left[\frac{D_i}{Q_i} C_{oi} + \frac{Q_i}{2} C_{hi}\right]$

Subject to $\sum_{i=1}^{n} f_i Q_i \leq W$, and all $Q_i \geq 0$.

Step (i) Using Khun-Tucker necessary condition, using Langragne multiplier $\lambda = 1$, we get,

Optimal orderable quantity for each item, $Q_i^* = \sqrt{\frac{2D_iC_{oi}}{C_{hi} + 2\lambda f_i}}$

Step (ii) If $\sum Q_i^* \cdot fi$, is lower than the available floor area, then stops, otherwise goes to next step.

Step (iii) Increase the value of λ, and follow iterations and find out optimum quantities.

SOLVED PROBLEM

Problem 6.22: *A shopkeeper keeping three different kinds of grains but he is bound to keep the stock which can be accommodated in the go down having space of 350 sq. mtr. Suggest him optimum order quantity according to its storage space. Take carrying cost is 20% of cost per unit.*

Products	*I*	*II*	*III*
Demand Rate (tons/year)	*10,000*	*30,000*	*45,000*
Ordering Cost (Rs. Per order)	*20*	*10*	*40*
Cost (Rs./ton)	*180*	*120*	*220*
Space requirement (Sq. Mtr.)	*0.4*	*0.7*	*0.8*

Solution:

Using Khun-Tucker conditions, we find the optimum quantity of each tool:

Optimum batch quantity; $Q_i^* = \sqrt{\frac{2D_iC_{oi}}{C_{hi}+2\lambda f_i}}$. Now, take $\lambda = 1$;

$$Q_1^* = \sqrt{\frac{2D_1C_{o1}}{C_{h1}+2\lambda f_1}} = \sqrt{\frac{2\,(10{,}000)(20)}{180\cdot(.2)+2\,(1)(0.4)}} = 104.25 \text{ tons}$$

$$Q_2^* = \sqrt{\frac{2D_2C_{o2}}{C_{h2}+2\lambda f_2}} = \sqrt{\frac{2\,(30{,}000)(10)}{120\cdot(.2)+2\,(1)(0.7)}} = 153.69 \text{ tons}$$

$$Q_3^* = \sqrt{\frac{2D_3C_{o3}}{C_{h3}+2\lambda f_3}} = \sqrt{\frac{2\,(45{,}000)(40)}{220\cdot(.2)+2\,(1)(0.8)}} = 280.98 \text{ tons}$$

Now, calculating the total floor space, = 104.25 *0.4 + 153.69 * 0.7 + 280.98 * 0.8

= 374.069 sq. mtr (> 75 sq. mtr.)

So, by changing the value of λ and by trial and error method, if we take, we take, $\lambda = 4.75$ we get $Q_1^* = 100.25$ units; $Q_2^* = 139.91$ units; $Q_3^* = 264.13$ units, we get investment is average inventory level in units = 349.34 (~ 350 sq. mtr.).

PRACTICE PROBLEM

EXERCISE 6.20: A mini factory unit produces three products. The factory premise is only 650 sq. ft of storage space. The shop uses an inventory holding charge equal to 20% of avg. Inventory valuation per year. Determine the optimal lot size for each item assuming no shortage is allowed based on following available data from the factory:

Products	I	II	III
Demand Rate (unit/year)	10,000	5,000	2,000
Ordering Cost (Rs. Per order)	75	100	200
Unit Cost (Rs.)	5	10	15
Floor space required (sq ft/unit)	0.40	0.7	0.8

6.6.3.2 EOQ model with investment constraints.

Suppose, C_i is the capital required for each item, then their total must be less than the total available fund (F), say F. It means that even all items are ordered, even than fund must be sufficient to order all. So, TVC minimum will depends on two factors: Ordering cost and inventory holding costs.

So, we can write, min TVC = $\sum_{i=1}^{n}\left[\frac{D_i}{Q_i} C_{oi} + \frac{Q_i}{2} C_{hi}\right]$

Subject to $\sum_{i=1}^{n} C_i Q_i \leq$ F, and all $Q_i \geq 0$.

Step (i) Using Khun-Tucker necessary condition, using Langragne multiplier $\lambda = 1$, we get,

Optimal orderable quantity for each item, $Q_i^* = \sqrt{\frac{2D_iC_{oi}}{C_{hi} + 2\lambda C_i}}$, C_{hi} = r X Ci .

Step (ii) If $\sum Q_i^* \cdot Ci$, is lower than the available fund, then stops, otherwise goes to next step.

Step (iii) Increase the value of λ, and follow iterations and find out optimum quantities.

SOLVED PROBLEM

***Problem 6.23:** A shop is producing different tools but it has constraint of investment that it cannot go beyond Rs. 1,000 as average inventory carrying cost. Based on following data find out the optimum quantity for each tool, keeping no shortage is allowed.*

Products	*I*	*II*	*III*
Demand Rate (unit/year)	*10,000*	*12,000*	*7,500*
Set-up Cost (Rs. Per order)	*50*	*40*	*60*
Unit Cost (Rs.)	*6*	*7*	*5*
Carrying cost (Rs./unit/year)	*20*	*20*	*20*

Solution:

Using Khun-Tucker conditions, we find the optimum quantity of each tool:

Optimum batch quantity; $Q_i^* = \sqrt{\frac{2D_iC_{oi}}{C_{hi}+2\lambda C_i}}$. Now, take $\lambda = 1$;

$$Q_1^* = \sqrt{\frac{2D_1C_{o1}}{C_{h1}+2\lambda C_1}} = \sqrt{\frac{2\,(10{,}000)(50)}{20+2\,(1)(6)}} = 177 \text{ units}$$

$$Q_2^* = \sqrt{\frac{2D_2C_{o2}}{C_{h2}+2\lambda C_2}} = \sqrt{\frac{2\,(12{,}000)(40)}{20+2\,(1)(7)}} = 168 \text{ units}$$

$$Q_3^* = \sqrt{\frac{2D_3C_{o3}}{C_{h3}+2\lambda C_3}} = \sqrt{\frac{2\,(7{,}500)(60)}{20+2\,(1)(5)}} = 173 \text{ units}$$

Now, calculating the average inventory cost,

$$= \sum_{i=1}^{3} C_i \cdot \frac{Q_i}{2} = 6\left(\frac{177}{2}\right) + 7\left(\frac{168}{2}\right) + 5\left(\frac{173}{2}\right) = \text{Rs. } 1{,}551 > 1{,}000.$$

So, now changing the value of $\lambda = 4, Q_1^* = 121$ units; $Q_2^* = 112$ units; $Q_3^* = 123$ units but still the investment is Rs. 1,112. 5.

So, we get, by trial and error method, if we take, we take $\lambda = 4.7$, we get $Q_1^* = 114$ units; $Q_2^* = 105$ units; $Q_3^* = 116$ units, we get investment is Rs. 999.5.

PRACTICE PROBLEM

EXERCISE 6.21: A company wants to determine the economic order quantity for each of its two items which are stored in its warehouse based on following data. The company wants to limits its inventory holding cost to Rs. 4000.

Products	I	II
Demand Rate (unit/year)	12,000	18,000
Ordering Cost (Rs. Per order)	500	400
Carrying Unit Cost (Rs.)	5	8

6.6.3.3 EOQ model with average inventory constraints.

Since, average number of units in inventory of an i item is $\frac{Q_i}{2}$, and the maximum inventory can be kept is M, then the sum of all average inventories must be less than M. So, for such conditions, minimum TVC, $= \sum_{i=1}^{n}\left[\frac{D_i}{Q_i} C_{oi} + \frac{Q_i}{2} C_{hi}\right]$

Subject to $\frac{1}{2}\sum_{i=1}^{n} Q_i \leq$ M, and all $Q_i \geq 0$.

Step (i) Using Khun-Tucker necessary condition, using Langrangian multiplier $\lambda = 1$, we get,

Optimal orderable quantity for each item, $Q_i^* = \sqrt{\frac{2D_iC_{oi}}{C_{hi} + 2\lambda}}$.

Step (ii) If sum of all average inventories is lower than the total maximum inventories, then stops, otherwise goes to next step.

Step (iii) Increase the value of λ, and follow iterations and find out optimum quantities.

SOLVED PROBLEM

Problem 6.24: ***From the following data find out the economic batch quantity if the average inventory level is restricted to 700 units of all items. Carrying cost is 20% for all products.***

Products	*I*	*II*	*III*
Demand Rate (unit/year)	*10,000*	*5,000*	*2,000*
Ordering Cost (Rs. Per order)	*75*	*100*	*200*
Unit Cost (Rs.)	*5*	*10*	*15*

Solution:

Using Khun-Tucker conditions, we find the optimum quantity of each tool:

Optimum batch quantity; $Q_i^* = \sqrt{\frac{2D_iC_{oi}}{C_{hi}+2\lambda_i}}$. Now, take $\lambda = 1$;

$$Q_1^* = \sqrt{\frac{2D_1C_{o1}}{C_{h1}+2\lambda}} = \sqrt{\frac{2\,(10{,}000)(75)}{0.20*5+2(1)}} = 707 \text{ units}$$

$$Q_2^* = \sqrt{\frac{2D_2C_{o2}}{C_{h2}+2\lambda}} = \sqrt{\frac{2\,(5{,}000)(100)}{0.20*10+2(1)}} = 500 \text{ units}$$

$$Q_3^* = \sqrt{\frac{2D_3C_{o3}}{C_{h3}+2\lambda}} = \sqrt{\frac{2\,(2{,}000)(200)}{0.20*15+2(1)}} = 400 \text{ unit.}$$

Now, calculating the average inventory items,

$$= \sum_{i=1}^{3} C_i \cdot \frac{Q_i}{2} = \left(\frac{707}{2}\right) + \left(\frac{500}{2}\right) + \left(\frac{400}{2}\right) = \text{Rs. } 803 > 700.$$

So, by changing the value of λ and by trial and error method, if we take, we take, $\lambda = 1.58$, we get $Q_1^* = 600$ units; $Q_2^* = 440$ units; $Q_3^* = 362$ units, we get investment is average inventory level in units = 700 units.

PRACTICE PROBLEM

EXERCISE 6.22: From the following data find out the economic order quantity if the average inventory level is restricted to 200 units of all items. Assume inventory holding cost as 10% of average inventory value.

Products	I	II	III
Demand Rate (unit/year)	5,000	4,000	3,000
Ordering Cost (Rs. Per	25	21	27
Unit Cost (Rs.)	10	12	16

6.6.3.4 EOQ model with number of order constraints.

It is used to find EOQ in case of multi-item inventory problems with a constraint on the number of orders to be placed per year. Using certain assumptions, such as:

- Ordering cost is same for all items.
- Demand is constant.
- An order is received in lots.
- No shortage is allowed.

Let's assume that, DC = demand in Rupees an N= Number of orders.

So, Number of orders per year, $= N \times \frac{\sqrt{DC}}{\Sigma\sqrt{DC}}$

SOLVED PROBLEM

Problem 6.25: From the following data, find out the number of each item's order such that total number of orders is restricted to 30.

Products	*A*	*B*	*C*	*D*
Demand Rate (unit/year)	*25,000*	*39,000*	*18,000*	*6,800*
Unit Cost (Rs.)	*5*	*8*	*10*	*12*
Ordering Cost (Rs.)	*25*	*23*	*55*	*15*

Solution:

For this kind of problem, where number of orders are fixed, than we calculate each products' total cost and then calculate each product's contribution in total cost. Based on its ordering cost, we calculate the number of orders in each product as given below:

Products	A	B	C	D	Total
Demand Rate (unit/year)	25,000	39,000	18,000	6,800	
Unit Cost (Rs.)	5	8	10	12	
Ordering Cost (Rs.)	25	23	55	15	
$\sqrt{D \cdot C}$	354	559	424	286	1623
Contribution of each product	0.218	0.344	0.261	0.177	1
Number of orders	0.218 (25) = 5	0.344 (23) = 8	0.261 (55) = 14	0.177 (15) = 3	30

PRACTICE PROBLEM

EXERCISE 6.23: From the following data, find out the number of each item's order such that total number of orders is restricted to 23.

Products	A	B	C
Demand Rate (unit/year)	30,000	20,000	10,000
Unit Cost (Rs.)	25	31	20
Ordering Cost (Rs.)	20	25	27

6.6.4. Single Item with Quantity Discounts

As we know that when any item is purchased in bulk, quantity discount is given to the buyer to encourage to buy more items. Although on first look, discount may look attractive, but it is necessary to find out trade-off between actual savings based on the purchasing cost, inventory holding cost and ordering cost.

Here, we assume that the demand is known and constant and not permitting any shortage. Also, we assume that replenishment is instantaneous.

Generally quantity discounts are given in two ways:

1. All units are quantity discounted.
2. Incremental quantity discount is given.

6.6.4.1 EOQ model with all units discounts.

Here, discount price is applicable to all units. So, the TVC is depending upon purchasing price, ordering cost and holding cost.

$$\therefore TC = DC_i + \frac{D}{Q} C_0 + \frac{D}{2} C_h$$

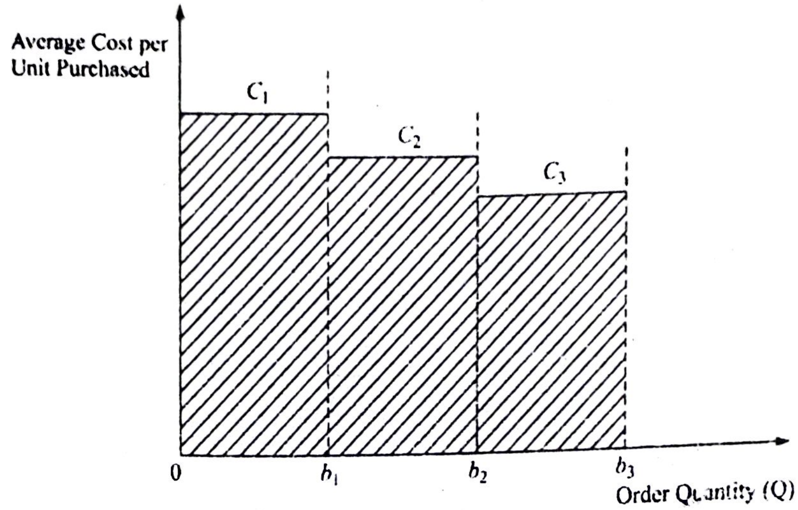

Figure 6.9 EOQ model with all units discounts

Since, the total cost curve is discontinuous; the calculus method for finding TC is not useable. So, easy method for finding out TC is as below:

TC_i = Total purchase cost of an item + Total variable inventory cost

$= DC_i + TVC \qquad = DC_i + \sqrt{2DC_o(rC_i)}$

Case I: Model with One - price break

Suppose, the following is the price discount given for specified quantity as below:

Quantity	*Price per unit (Rs.)*
$0 \leq Q_1 < b_1$	C_1
$b_1 \leq Q_2$	C_2 ($C_2 < C_1$)

Steps to be followed:

Step 1: Consider the lowest price (*i.e.,* C_2) and determine Q_2^* by using EOQ model,

$$Q_2^* = \sqrt{\frac{2DC_o}{rC_2}}$$

If Q_2^* is greater than b_1, than order Q_2^* as optimum order quantity. At that time, the optimal total cost,

$$TC^* = DC_2 + \frac{D}{b_1} C_o + \frac{b_1}{2} (C_2 \, X \, r)$$

Step 2: If Q_2^* is not equal or more than b_1, then calculate optimal order quantity, using C_1 as price and find corresponding TC at that quantity.

If TC (b_1) > TC (Q^*_1), then EOQ = $Q^* = Q^*_1$, otherwise, $Q^* = b_1$.

SOLVED PROBLEM

Problem 6.26: ***Find the optimum quantity for a product each cost Rs. 25 and monthly demand is 1,800 units. The cost per order is Rs. 150 and the inventory carrying charges work out to 20% of the average inventory. Would you accept a 2.5% discount on a minimum supply quantity of 1,250 units?***

Solution:

Here, D = 21,600 per annum; C= 25; Co = Rs. 150 per order; r = 20%.

$$Q^* = \sqrt{\frac{2DC_0}{cr}} = \sqrt{\frac{2(21600)(150)}{25\,(0.2)}} = 1{,}138 \text{ units.}$$

$$TC(Q^*) = DC + \frac{D}{Q^*}C_o + \frac{Q^*}{2}(C\,X\,r) \qquad = 21{,}600 \text{ X } 25 + \frac{(21600)(150)}{1138} + \frac{1138}{2}(25 * 0.2)$$

$$= \text{Rs. } 5,\,45{,}692.$$

When 2.5% is offered the minimum number of units is 1,250, then price of an item is 24.375.

$$\text{Then, } TC(Q^*) = DC + \frac{D}{Q^*}C_o + \frac{Q^*}{2}(C\,X\,r) = 21{,}600 \text{ X } 24.375 + \frac{(21600)(150)}{1250} + \frac{1250}{2}(25 * 0.2)$$

$$= \text{Rs. } 5,\,32,\,217.$$

So, saving =Rs. 5, 45,692 – Rs. 5, 32,217 = Rs. 13, 475.

∴ The discount offer should be accepted.

Problem 6.27: ***The annual demand of a product is 15,000 units. Each unit costs Rs. 50 if the orders are placed in quantities below 150 units. For order of 200 or above, the unit price is Rs. 44. Assume inventory holding cost is 12% if the value of the item and the ordering cost is Rs. 2 per order. Find the economic lot size. (GTU – May 2014)***

Solution:

Here, D = 15,000 units; b_1 = 150 units; C_o = Rs. 2 per order; r = 12 % of price of an item.

The unit cost for the range of quantities is as below:

Quantity	Price per unit (Rs.)
$0 \le Q_1 < 150$	50
$150 \le Q_2$	44

Step 1: Consider the lowest price (*i.e.*, C_2) and determine Q_2^* by using EOQ model,

$$Q_2^* = \sqrt{\frac{2DC_o}{rC_2}} = \sqrt{\frac{2(15000)(2)}{0.12\,(44)}} = 106 \text{ units.}$$

Here, $Q_2^* \ngeq 150$ units. So, now calculating, Q_1^*.

$$Q_1^* = \sqrt{\frac{2DC_o}{rC_1}} = \sqrt{\frac{2(15000)(2)}{0.12\,(50)}} = 100 \text{ units.}$$

Now, comparing Total Cost at Q_1^* (= 100 units) and Total cost at b (= 150 units),

$$TC\,(Q_1^*) = DC_1 + \frac{D}{Q_1^*}C_o + \frac{Q_1^*}{2}(C_1 \, X\, r) = 15{,}000 * 50 + \frac{15000}{100} * 2 + \frac{100}{2}\,(50\; X\; 0.12)$$

$$= \text{Rs. } 7,\,50,\,600.$$

$$TC\,(b) = DC_2 + \frac{D}{b}C_o + \frac{b}{2}(C_2 \, X\, r) = 15{,}000 * 44 + \frac{15000}{150} * 2 + \frac{150}{2}\,(44\; X\; 0.12)$$

$$= \text{Rs. } 6,\,60,\,596.$$

Since, TC (b) < TC (Q_1^*), the optimal order quantity is b (= 150 units).

Case II: Model with two - price breaks

Suppose, the following is the price discount given for specified quantity as below:

Quantity	Price per unit (Rs.)
$0 \le Q_1 < b_1$	C_1
$b_1 \le Q_2 < b_2$	C_2 ($C_2 < C_1$)
$b_2 \le Q_2 < b_3$	C_3 ($C_3 < C_2$)

Steps to be followed:

Step 1: Consider the lowest price and calculate Q_3^* by using EOQ formula.

Step 2: If $Q_3 \ge b_2$, then Q_3^* is the optimum quantity and calculate the corresponding TC*. Otherwise go to step 3.

Step 3: Consider the second lowest price and calculate Q_2^* by using EOQ formula.

Step 4: If $Q_2 \ge b_1$ then Q_2^* is the optimum quantity and calculate the corresponding TC*. Otherwise go to step 5.

Step 5: If Q_2^* is not equal or more than b_1, then calculate optimal order quantity, using C_1 as price and find corresponding TC at that quantity.

If TC (b_1) > TC (Q^*_1), then EOQ = $Q^* = Q^*_1$, otherwise, $Q^* = b_1$.

Problem 6.28: ***Find the optimum quantity for a product each cost Rs. 18 and annual demand is 2,700 units. The cost per order is Rs. 40 and the inventory carrying charges work out to 25% of the average inventory. Further, suppose the supplier offers a 10% discount on orders between 400 and 699 units, and a 20% discount on orders exceeding or equal to 700. Should the advantage of discount can be taken?***

Solution:

Here, d = 2,700 units; C = Rs. 18 per unit;

C_o = Rs. 40 per order; $r = 25\% \therefore C_h = 18 \text{ X } 0.25$ = Rs. 4.50.

$$Q^* = \sqrt{\frac{2DC_o}{cr}} = \sqrt{\frac{2(2700)(40)}{4.5}} = 219 \text{ units.}$$

$$TC(Q^*) = DC + \frac{D}{Q*}C_o + \frac{Q*}{2}(C \, X \, r) = 2,700 \text{ X } 18 + \frac{(2700)(40)}{219} + \frac{219}{2}(4.5)$$

$$= \text{Rs. } 49,586.$$

When quantity discount are offered, the following information is to be used:

Quantity	Price per unit (Rs.)
$0 \le Q_1 < 399$	18
$400 \le Q_2 < 699$	16.2
$700 \le Q_2$	14.4

$$Q_3^* = \sqrt{\frac{2DC_o}{cr}} = \sqrt{\frac{2(2700)(40)}{16.2\,(0.25)}} = 231 \text{ units.}$$

Since, the Q_3^* lies in the first range, we need to compare TC (Q*); TC (b_1) and TC (b_2) with each other.

$$TC(b_1) = DC + \frac{D}{Q*}C_o + \frac{Q*}{2}(C \, X \, r)$$

$$= 2,700 \text{ X } 16.2 + \frac{(2700)(40)}{400} + \frac{400}{2}(4.05) \quad = \text{Rs. } 44,820.$$

$$TC(b_2) = DC + \frac{D}{Q*}C_o + \frac{Q*}{2}(C \, X \, r)$$

$$= 2,700 \text{ X } 14.4 + \frac{(2700)(40)}{700} + \frac{700}{2}(3.6) \quad = \text{Rs. } 40,294.$$

Since, the total cost at 700 units as order quantity, is least, the discount offer should be accepted.

Problem 6.29: ***The annual demand of a product is 15,000 units. Each unit costs Rs. 50 if the orders are placed in quantities below 150 units. Assume inventory holding cost is***

12% if the value of the item and the ordering cost is Rs. 2 per order. For order of 200 or above, the unit price is Rs. 45. Also, further discounting price Rs. 42 is offered, for order of 500 units or above. Should the discounted offer be accepted?

Solution:

Here, d = 15,000 units; C = Rs. 50 per unit;

C_o = Rs. 2 per order; r = 12% ∴ C_h = 50 X .12 = Rs. 6.

b_1 = 200 units; b_2 = 500 units

$$Q^* = \sqrt{\frac{2DC_o}{cr}} = \sqrt{\frac{2(15000)(2)}{6}} = 100 \text{ units.}$$

$$TC\ (Q^*) = DC + \frac{D}{Q*}C_o + \frac{Q*}{2}(C\ X\ r) = 15,000\ X\ 50 + \frac{(15000)(2)}{100} + \frac{100}{2}(6) = \text{Rs. } 7,50,600.$$

When quantity discount are offered, the following information is to be used:

Quantity	***Price per unit (Rs.)***
$0 \le Q_1 < 200$	50
$200 \le Q_2 < 499$	45
$500 \le Q_2$	42

$$Q_3^* = \sqrt{\frac{2DC_o}{cr}} = \sqrt{\frac{2(15000)(2)}{45\ (0.12)}} = 106 \text{ units.}$$

Since, the Q_3* lies in the first range, we need to compare TC (Q*); TC (b_1) and TC (b_2) with each other.

$$TC\ (b_1) = DC + \frac{D}{Q*}C_o + \frac{Q*}{2}(C\ X\ r)$$

$$= 15,000\ X\ 45 + \frac{(15000)(2)}{200} + \frac{200}{2}(5.4)$$

$$= \text{Rs. } 6,75,840.$$

$$TC\ (b_2) = DC + \frac{D}{Q*}C_o + \frac{Q*}{2}(C\ X\ r)$$

$$= 15,000\ X\ 42 + \frac{(15000)(2)}{500} + \frac{500}{2}(5.04) = \text{Rs. } 6,31,320.$$

Since, the total cost at 500 units as order quantity, is least, the discount offer should be accepted.

Problem 6.30: Find the optimal order quantity for a product for which the price breaks are as follows:

Quantity	*Price per unit (Rs.)*
$0 \le Q_1 < 500$	1000
$500 \le Q_2 < 4000$	925
$4000 \le Q_2$	875

Order cost = Rs. 350/-; Demand = 2400; time period = 360 days; cost of carrying = 6%.

Solution:

Here, d = 2400 units; C = Rs. 1000 per unit;

C_o = Rs. 350 per order; r = 6% ∴ $C_h = 1000 \times 0.06$ = Rs. 60.

b_1 = 500 units; b_2 = 4000 units

$$Q^* = \sqrt{\frac{2DC_o}{cr}} = \sqrt{\frac{2(2400)(350)}{60}} = 168 \text{ units.}$$

$$TC(Q^*) = DC + \frac{D}{Q^*}C_o + \frac{Q^*}{2}(C \times r) \qquad = 2400 \times 1000 + \frac{(2400)(350)}{168} + \frac{168}{2}(60)$$

$$= \text{Rs. } 24,10,040.$$

When quantity discount are offered, the following information is to be used:

Quantity	*Price per unit (Rs.)*
$0 \le Q_1 < 500$	1000
$500 \le Q_2 < 4000$	925
$4000 \le Q_2$	875

$$Q_3^* = \sqrt{\frac{2DC_o}{cr}} = \sqrt{\frac{2(2400)(350)}{925\,(0.06)}} = 174 \text{ units.}$$

Since, the Q_3^* lies in the first range, we need to compare TC (Q^*); TC (b_1) and TC (b_2) with each other.

$$TC(b_1) = DC + \frac{D}{Q^*}C_o + \frac{Q^*}{2}(C \times r)$$

$$= 2400 \times 925 + \frac{(2400)(350)}{500} + \frac{500}{2}(55.5) \quad = \text{Rs. } 22,35,555$$

$$TC(b_2) = DC + \frac{D}{Q^*}C_o + \frac{Q^*}{2}(C \times r)$$

$$= 2400 \times 875 + \frac{(2400)(350)}{4000} + \frac{4000}{2}(52.5) \quad = \text{Rs. } 22,05,210$$

Since, the total cost at 500 units as order quantity, is least, the discount offer should be accepted.

Problem 6.31: The annual demand of a product is 10000 units. Each unit costs Rs. 50 if the orders are placed in quantities below 140 units. For order of 180 or above, the

unit price is Rs. 44. Assume inventory holding cost is 10% if the value of the item and the ordering cost is Rs. 2 per order. Find the economic lot size. ***(GTU – Nov. 2016)***

Solution:

Here, $D = 10000$ units; $b_1 = 140$ units; $C_o =$ Rs. 2 per order; $r = 10$ % of price of an item.

The unit cost for the range of quantities is as below:

Quantity	Price per unit (Rs.)
$0 \leq Q_1 < 140$	50
$140 \leq Q_2$	44

Step 1: Consider the lowest price (*i.e.*, C_2) and determine Q_2^* by using EOQ model,

$$Q_2^* = \sqrt{\frac{2DC_o}{rC_2}} = \sqrt{\frac{2(10000)(2)}{0.10\ (44)}} = 95.34 \text{ units.}$$

Here, $Q_2^* \ngeq 140$ units. So, now calculating, Q_1^*.

$$Q_1^* = \sqrt{\frac{2DC_o}{rC_1}} = \sqrt{\frac{2(10000)(2)}{0.10\ (50)}} = 89.44 \text{ units.}$$

Now, comparing Total Cost at Q_1^* (= 90 units) and Total cost at b (= 140 units),

$$TC\ (Q_1^*) = DC_1 + \frac{D}{Q_1^*}C_o + \frac{Q_1^*}{2}(C_1 X r) = 10{,}000 * 50 + \frac{15000}{90} * 2 + \frac{90}{2}(50\ X\ 0.10)$$

$$= \text{Rs. } 500447.22$$

$$TC\ (b) = DC_2 + \frac{D}{b}C_o + \frac{b}{2}(C_2 X r) = 10{,}000 * 44 + \frac{10000}{140} * 2 + \frac{140}{2}(44\ X\ 0.10)$$

$$= \text{Rs. } 440450.85$$

Since, TC (b) < TC (Q_1^*), the optimal order quantity is b (= 140 units).

PRACTICE PROBLEMS

EXERCISE 6.24: An automobile manufacturer purchases 2,400 castings over a period of 360 days each costs Rs. 1000. The ordering cost is Rs. 70,000 per order and the storage cost is 0.12% of unit cost. Determine the optimal purchase quantity if the supplier has offered the 50 Rs. discount if the ordered quantity is equal of more than 1000 units.

EXERCISE 6.25: An automobile manufacturer purchases 2,400 castings over a period of 12 months each costs Rs. 10. The ordering cost is Rs. 350 per order and the storage cost is 2% of unit cost. Determine the optimal purchase quantity if the supplier has offered the 0.75 Rs. discount if the ordered quantity is equal of more than 500 units.

EXERCISE 6.26: The annual demand of a product is 10, 000 units. Each unit cost Rs. 100 if orders are placed in quantities below 200 units but for orders of 200 or above the price is Rs. 95. The annual inventory holding is 10% of the value of the item and the ordering cost is Rs. 5 per order. Should the discounting offer is to be accepted?

EXERCISE 6.27: If the monthly demand is 200 units, storage cost is 2% of the unit cost and ordering cost is Rs. 100, then for the following given data find out optimum order quantity:

Quantity	Price per unit (Rs.)
$0 \leq Q_1 < 499$	10
$500 \leq Q_2 < 750$	9.25
$750 \leq Q_2$	8.7

EXERCISE 6.28: If the monthly demand is 500 units, storage cost is 10% of the unit cost and ordering cost is Rs. 180, then for the following given data find out optimum order quantity:

Quantity	Price per unit (Rs.)
$0 \leq Q_1 < 499$	25
$500 \leq Q_2 < 1500$	24.8
$1500 \leq Q_2$	24.6

EXERCISE 6.29: If the monthly demand is 200 units, storage cost is 25% of the unit cost and ordering cost is Rs. 20, then for the following given data find out optimum order quantity:

Quantity	Price per unit (Rs.)
$0 \leq Q_1 < 49$	10
$50 \leq Q_2 < 100$	9
$100 \leq Q_2$	8

6.7 ABC Analysis (GTU – Dec. 2012; Nov - 2014)

In the organizations, there are thousands of items having different unit costs, inventory handling costs, shortage costs, lead time as well as usage costs. It is practically impossible to keep an eye on each kind of item and its stock. So, some management of inventory has to be done systematically such that inventory need can be fulfilled.

The management has to pay more attention to items whose usage value is high and less attention to items having low usage value.

ABC analysis is one inventory management technique which controls the inventory in the organization very efficiently. In this analysis inventory is classified in three categories based on their usage value.

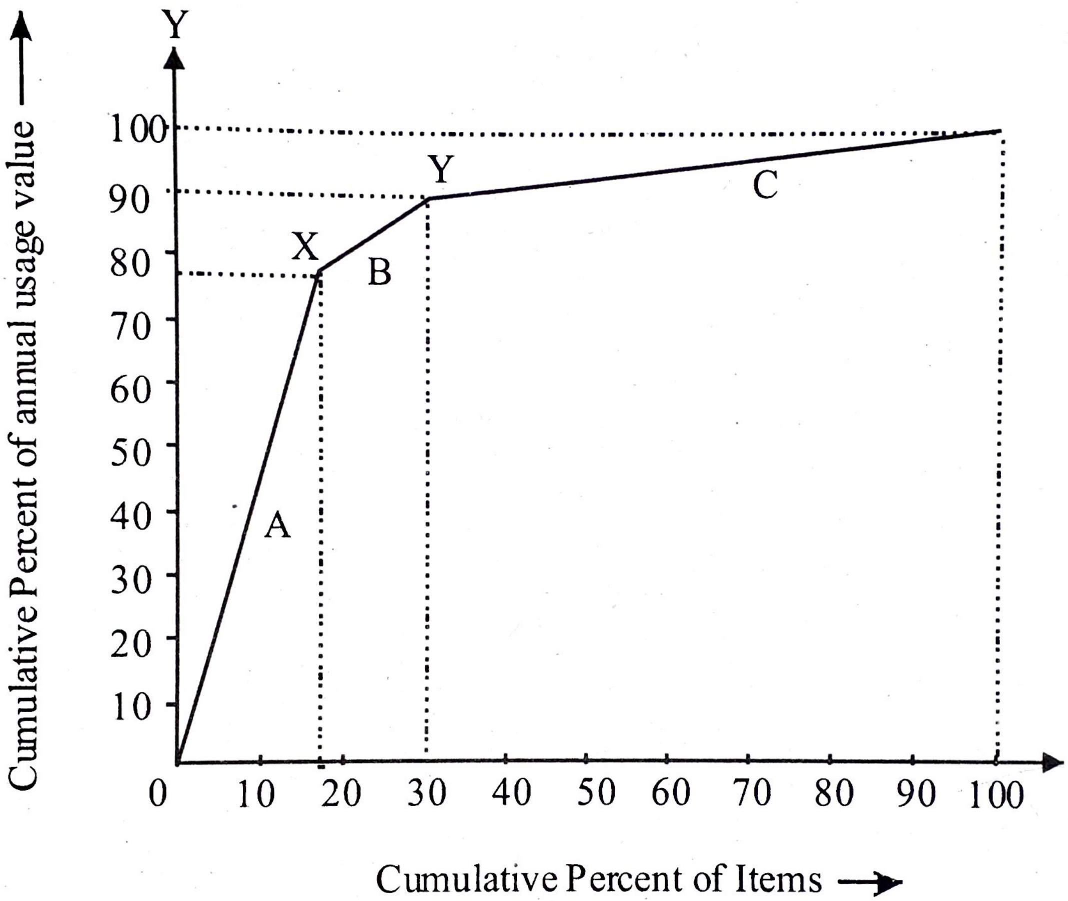

Figure 6.10 ABC Analysis

Class A. Those inventories accounting 70 to 80 % of annual usage value and its quantity are 5 to 10% of total number of inventories.

Class B. Those inventories accounting 15 to 20 % of annual usage value and its quantity is 10 to 20% of total number of inventories.

Class C. Those inventories accounting 5 to 15 % of annual usage value and its quantity is 70 to 80% of total number of inventories.

Following steps are performed for the ABC analysis:

Step 1: Find the annual usage value of every item in the sample by multiplying the annual requirement by its unit cost.

Step 2: Arrange these items in descending order of usage value computed above.

Step 3: Accumulate the total number of items and the usage value.

Step 4: Convert the accumulated totals of number of items and usage value into percentage of the grand totals.

Step 5: Plot these two percentages on the graph. On X axis put cumulative percentage of items and on Y axis put cumulative of annual usage value.

Step 6: Mark cut-off points X and Y where the curve changes its slope. Divided curve will give three segments A, B and C.

Under ABC analysis, an organization would devote much time and effort in controlling of 'A' items. Extra care will be taken for determining the minimum, maximum inventory and reorder level etc. of the 'A' items whereas least control is need on the item 'C'.

'A' items is including high value items, so fixed order quantity method is used for ordering the items. 'C' items having low value items, so there are followed by bulk purchase orders. 'B' items are usually placed under statistical control and to attract periodic control of the management.

Advantages of ABC analysis:

1. ABC analysis is dynamic and efficient inventory management.
2. It provides more sound cost perspective and helps in achieving cost reduction by controlling inventories.
3. It avoids wastage of time and energy.
4. It is used in controlling raw-materials, components and WIP.
5. It achieves control over selected items.